



COMPENSATION OF SPACE-CHARGE MISMATCH AT
TRANSITION OF BOOSTER USING
THE TRANSITION-JUMP METHOD

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This method was applied successfully¹ on the CERN PS. Its application to the NAL accelerators was studied by E. Courant.² A theoretical analysis of this method was made by D. Möhl.³ A more detailed design of such a system for the NAL booster synchrotron is given here based on Möhl's analysis.

Möhl's Analysis

The qualitative and approximate quantitative results of Möhl's analysis are summarized below.

1. The transition-jump method is effective to partially compensate for the phase-space mismatch caused by space charge at transition which becomes prominent for $\eta_0(0) \gg 1$. For best overall matching one needs a delayed jump which starts near transition ($x_1 \sim 0$ in Möhl's notation). This and $\eta_0(0) \gg 1$ are assumed for all subsequent formulas.

2. Bunch-length matching for an infinitely fast jump gives a lower limit for the magnitude of the jump

$$\frac{\delta\gamma_t}{\delta\gamma} \gtrsim 1.5 (\eta_0(0))^{4/3} \quad (1)$$

where $\delta\gamma_t$ is the jump of the transition γ , and $\delta\gamma \equiv \dot{\gamma} T$ (T = time interval about transition in which the adiabatic approximation is



not valid). $\delta\gamma$ is, therefore, the range of γ about transition in which the adiabatic approximation is not valid.

3. For finite jump speed $(|\dot{\gamma}_t| \neq \infty)$ the increase in phase area A is given by $(\frac{\delta A}{A} \ll 1)$

$$\frac{\delta A}{A} \sim \frac{0.60}{(\eta_o(0))^{2/3}} \frac{\left(\frac{\delta\gamma_t}{\delta\gamma}\right)^2}{\left(1 + \frac{|\dot{\gamma}_t|}{\dot{\gamma}}\right)^{4/3}} \quad (2)$$

which gives the desired jump speed $\dot{\gamma}_t$. If $\frac{\delta\gamma_t}{\delta\gamma}$ is given by Equ. (1) we have

$$\frac{\delta A}{A} \sim 1.4 \frac{(\eta_o(0))^2}{\left(1 + \frac{|\dot{\gamma}_t|}{\dot{\gamma}}\right)^{4/3}} \quad (3)$$

4. The growth in $\left(\frac{\Delta p}{p}\right)_{\max} \equiv \Delta$ is given by

$$\frac{\Delta}{\Delta_o} \sim (\eta_o(0))^{-1/3} \left[1 + 0.63 \eta_o(0) \frac{\frac{\delta\gamma_t}{\delta\gamma}}{\left(1 + \frac{|\dot{\gamma}_t|}{\dot{\gamma}}\right)^{2/3}} \right] \quad (4)$$

where Δ_o is the $\frac{\Delta p}{p}$ at transition in the absence of space charge.

If $\frac{\delta\gamma_t}{\delta\gamma}$ is given by Equ. (1) we have

$$\frac{\Delta}{\Delta_o} \sim (\eta_o(0))^{-1/3} + 0.95 \frac{(\eta_o(0))^2}{\left(1 + \frac{|\dot{\gamma}_t|}{\dot{\gamma}}\right)^{2/3}} \quad (5)$$

Booster Parameters

The transition parameters $\eta_o(0)$ and T can be given in terms of dimensionless parameters by⁴

$$\eta_o(0) = \frac{\sqrt{3} \pi^{7/2}}{2 (\Gamma(2/3))^3} \frac{r_p}{R} \frac{Ng_o}{A^{3/2}} \left(\frac{f_{rf}}{\dot{\gamma}} \right)^{1/2} \quad (6)$$

$$T^3 = \frac{1}{4\pi} \frac{\gamma^4}{f_{rf} \dot{\gamma}^2 |\cot \phi_s|} \quad (7)$$

where for the booster at transition the parameters are

$\Gamma(2/3)$ = factorial function of $2/3 = 1.354$

r_p = classical proton radius = 1.53×10^{-18} m

R = machine radius = 75.47 m

N = total number of protons per pulse = 3.5×10^{12}

g_o = geometric factor ~ 4.5

f_{rf} = rf frequency = $52.20 \times 10^6 \text{ sec}^{-1}$

$\dot{\gamma} \equiv$ time rate of increase of $\gamma = 0.407 \times 10^3 \text{ sec}^{-1}$

ϕ_s = synchronous phase = $90^\circ \pm 20^\circ$

A = total phase area occupied by beam in $\frac{p}{mc} \phi$

units ($A = \frac{hS}{mcR}$ where S = phase area of each

beam bunch in $w\phi$ units, and $dw = \frac{R}{h} dp$) = 0.0069

With these parameters we have for the booster

$$\eta_o(0) = 3.8$$

$$T = 0.28 \times 10^{-3} \text{ sec}$$

$$\delta\gamma = \dot{\gamma}T = 0.114$$

and Equ. (1) gives

$$\boxed{\delta\gamma_t \sim 1.0} \quad (8)$$

For $\frac{|\dot{\gamma}_t|}{\dot{\gamma}} = 10$ and 20 , Equ. (3) gives $\frac{\delta A}{A} = 0.83$ and 0.35 . Therefore a jump speed of

$$\boxed{|\dot{\gamma}_t| \sim 10^4 \text{ sec}^{-1}} \quad (9)$$

or a jump time for $\delta\gamma_t = 1.0$ of 0.1 m sec is desirable. This jump speed gives $\frac{|\dot{\gamma}_t|}{\dot{\gamma}} \approx 25$ and

$$\boxed{\frac{\delta A}{A} \sim 0.26, \quad \frac{\Delta}{\Delta_0} \sim 2.2} \quad (10)$$

γ_t -jump Quadrupoles

Q-jump is a misleading name for the system. What one wants is a γ_t -jump system, and it is quite possible to shift γ_t by 1 unit without noticeably affecting v_x and v_y (or Q_x and Q_y). To see this we recall that the equation for the dispersion function x_p after Floquet transformation is

$$\frac{d^2 u}{d\phi^2} + v^2 u = \frac{(v\beta)^{3/2}}{R} \quad (11)$$

where

$$\begin{cases} u = \frac{x_p}{\sqrt{v\beta}} \\ d\phi = \frac{dz}{v\beta}, \quad z \text{ along orbit} \end{cases}$$

If we add to β a term with zero ϕ -average (say, $\frac{a}{v} \sin n\phi$), the betatron wave numbers v_x and v_y will not be affected. Neglecting other oscillatory terms in β and assuming $\frac{a}{R} \ll 1$ we can write approximately

$$\frac{1}{R} (v\beta)^{3/2} = \frac{1}{R} (R + a \sin n\phi)^{3/2} = \sqrt{R} + \frac{3}{2} \frac{a}{\sqrt{R}} \sin n\phi$$

The solution of Equ. (11) is then

$$u = \frac{\sqrt{R}}{v^2} + \frac{3}{2} \frac{\frac{a}{\sqrt{R}}}{v^2 - n^2} \sin n\phi$$

The change in orbit length per unit $\frac{\Delta p}{p}$ is

$$\begin{aligned} \delta L &= \oint \frac{x p}{R} dz = \oint \frac{1}{R} (v\beta)^{3/2} u d\phi \\ &= \oint \left[\frac{R}{v^2} + \frac{3}{2} a \left(\frac{1}{v^2} + \frac{1}{v^2 - n^2} \right) \sin n\phi \right. \\ &\quad \left. + \frac{9}{4} \frac{a^2}{R} \frac{1}{v^2 - n^2} \sin^2 n\phi \right] d\phi \\ &= \frac{2\pi R}{v^2} \left(1 + \frac{9}{8} \frac{a^2}{R^2} \frac{v^2}{v^2 - n^2} \right) \end{aligned}$$

The transition γ_t is, therefore, changed to

$$\gamma_t^2 = \frac{2\pi R}{\delta L} \approx v^2 - \frac{9}{8} \frac{a^2}{R^2} \frac{v^4}{v^2 - n^2} \quad (12)$$

If n is chosen to be the integer closest to v , because of the large factor $\frac{v^4}{v^2 - n^2}$ a small $\frac{a}{R}$ is adequate to reduce γ_t a great deal.

For the NAL booster the number of cells is $N = 24$ and ν (namely ν_x) is 6.7. We choose $n = 6$ because it is commensurable with N . Twelve 0.20 m long quadrupoles are placed in the mid-F positions (the cell structure is FOFDOOD) and evenly spaced around the ring. With the 12 quadrupoles excited alternately as F and D, computer runs using SYNCH give for various values of B' of the quadrupoles the following values of ν_x , ν_y , γ_t , and $x_{p \max}$ at transition.

$B' \text{ (kG/m)}$	ν_x	ν_y	$x_{p \max} \text{ (m)}$	γ_t
0	6.700	6.800	3.189	5.446
± 2.825	6.701	6.800	4.157	5.425
± 8.476	6.710	6.800	5.966	5.267
± 14.127	6.729	6.801	7.512	5.008
± 19.777	6.755	6.802	8.728	4.715
± 25.428	6.790	6.803	9.600	4.435
± 31.079	6.832	6.804	10.156	4.191

We see, therefore, with very modest quadrupole strength one can reduce γ_t by 1 unit. Of course in so doing $x_{p \max}$ is increased by a factor of 3. This increase is, of course, unavoidable but it is quite tolerable. The SYNCH output for the cases of $B' = 0$ and 25.428 kG/m are attached as an appendix.

Acknowledgment

Several interesting discussions with Lloyd Smith were very helpful. The SYNCH runs were made by G. Bellendir.

B' = 25.428 kg/m

BOOSTER TRANSITION JUMP

SYNCH RUN SYNC

corresponding values at transition given in parentheses

(25.428)
(167.543)

BETATRON FUNCTIONS THROUGH C = 1/2 OF RING

CYCLE	S	NAME	PSIN/2PI	BETAX	ALPHAX	KEQ	DREQ	NR	PSIN/2PI	MEYAT	ALPHAY	VEQ	DVEC	MY
1	0.0	HF	0.0	20.01089	-4.12780	-3.17636	-0.76019	4.47335	0.0	11.37857	2.23539	0.0	0.0	3.32321
2	0.01589	U	0.01589	35.94996	-0.53352	-4.18506	-0.96761	5.99583	0.06605	5.23730	0.19603	0.0	0.0	2.28851
3	0.03189	HF	0.03189	36.49241	-0.53139	-4.45236	-0.96761	6.04089	0.08149	5.09083	0.09690	0.0	0.0	2.25429
4	0.04789	U	0.04789	36.94241	0.53139	-4.45236	0.96761	6.04089	0.08775	5.09083	-0.09690	0.0	0.0	2.25429
5	0.06389	HF	0.06389	35.94996	0.53352	-4.41856	0.96761	5.99583	0.10318	5.23730	-0.19603	0.0	0.0	3.32321
6	0.07989	U	0.07989	20.01089	4.12780	-3.17636	-0.76019	4.47335	0.16924	11.37857	-2.23539	0.0	0.0	3.32321
7	0.09589	HF	0.09589	16.10846	3.61707	-2.79626	-0.76019	4.47335	0.17560	13.74572	-2.45891	0.0	0.0	3.32321
8	0.11189	U	0.11189	5.25677	0.66553	-1.03102	0.51100	2.29277	0.20005	22.03020	0.11310	0.0	0.0	4.49863
9	0.12789	HF	0.12789	1.15205	-0.98141	2.03500	0.51100	2.67433	0.24365	22.32801	-0.16274	0.0	0.0	4.49863
10	0.14389	U	0.14389	21.56109	-4.77577	4.23286	0.77838	5.61143	0.26761	14.12383	2.23539	0.0	0.0	3.32321
11	0.15989	HF	0.15989	26.59714	-5.31053	4.76872	0.77838	5.15724	0.27848	11.72875	2.26428	0.0	0.0	3.32321
12	0.17589	U	0.17589	46.47099	-0.48150	6.69372	0.21107	6.81696	0.33712	5.50239	0.20083	0.0	0.0	2.34572
13	0.19189	HF	0.19189	47.66475	-0.51331	6.94701	0.21107	6.60397	0.37281	5.29266	-0.02605	0.0	0.0	2.34572
14	0.20789	U	0.20789	31.48613	5.24409	8.09570	-0.77838	5.61143	0.44291	10.06943	-1.86514	0.0	0.0	3.17824
15	0.22389	HF	0.22389	26.47032	4.79153	7.99273	0.21579	5.14493	0.45014	12.04577	-2.08753	0.0	0.0	3.47070
16	0.23989	U	0.23989	12.78927	0.68331	9.52711	0.12201	5.76221	0.47848	18.76660	0.15203	0.0	0.0	4.13204
17	0.25589	HF	0.25589	8.71867	-0.00488	9.52711	0.12201	5.76221	0.50308	18.90487	-0.17508	0.0	0.0	4.14297
18	0.27189	U	0.27189	14.79800	-2.42793	7.38484	1.21579	3.84682	0.55811	12.22122	2.09987	0.0	0.0	3.49589
19	0.28789	HF	0.28789	17.34241	-2.66090	7.99273	1.21579	4.16442	0.56523	10.23201	1.87855	0.0	0.0	3.19875
20	0.30389	U	0.30389	25.06804	0.40157	9.60227	-0.14453	5.00800	0.63388	5.41573	0.06412	0.0	0.0	2.31102
21	0.31989	HF	0.31989	24.67806	0.37841	9.52800	-0.14453	4.96770	0.64857	5.41573	-0.06412	0.0	0.0	2.31102
22	0.33589	U	0.33589	24.67806	-0.37841	9.52800	0.14453	4.96770	0.65443	5.41573	0.06412	0.0	0.0	2.31102
23	0.35189	HF	0.35189	25.06804	-0.40157	9.60227	0.14453	5.00800	0.66912	10.23201	-1.87855	0.0	0.0	3.19875
24	0.36789	U	0.36789	17.34241	2.66090	7.99273	-1.21579	3.84682	0.71778	12.22122	-2.09987	0.0	0.0	3.49589
25	0.38389	HF	0.38389	14.79800	2.42793	7.38484	-1.21579	3.84682	0.74490	18.90487	0.17508	0.0	0.0	4.14297
26	0.39989	U	0.39989	8.71867	0.00488	9.52711	0.12201	5.76221	0.77292	18.90487	-0.17508	0.0	0.0	4.14297
27	0.41589	HF	0.41589	12.78927	-0.68331	9.52711	-0.12201	5.76221	0.82452	12.22122	2.09987	0.0	0.0	3.49589
28	0.43189	U	0.43189	26.47032	-4.79153	7.99273	1.21579	4.16442	0.85246	10.23201	-1.87855	0.0	0.0	3.19875
29	0.44789	HF	0.44789	31.48613	-5.24409	8.09570	-0.77838	5.61143	0.86009	10.06943	1.86514	0.0	0.0	3.17824
30	0.46389	U	0.46389	47.66475	0.51331	6.94701	0.21107	6.60397	0.90319	5.29266	-0.02605	0.0	0.0	2.34572
31	0.47989	HF	0.47989	46.47099	0.48150	6.69372	-0.21107	6.81696	0.94588	5.50239	0.20083	0.0	0.0	2.34572
32	0.49589	U	0.49589	26.59714	5.31053	4.76872	0.77838	5.15724	1.02921	11.72875	-2.26428	0.0	0.0	3.32321
33	0.51189	HF	0.51189	21.56109	4.77577	4.23286	-0.77838	5.61143	1.05935	14.12383	2.23539	0.0	0.0	3.32321
34	0.52789	U	0.52789	7.19205	0.98141	2.03500	-0.51100	2.67433	1.09325	22.32801	-0.16274	0.0	0.0	4.49863
35	0.54389	HF	0.54389	5.25677	-0.66553	-1.03102	0.51100	2.29277	1.10245	22.03020	0.11310	0.0	0.0	4.49863
36	0.55989	U	0.55989	16.10846	3.61707	-2.79626	-0.76019	4.47335	1.12740	13.74572	-2.45891	0.0	0.0	3.32321
37	0.57589	HF	0.57589	20.01089	-4.12780	-3.17636	-0.76019	4.47335	1.13377	11.37857	2.23539	0.0	0.0	3.32321

YHMS = 0.0

QY = 1.13340572

KHMS = 5.91279489

KHMS = 4.435177

QX = 1.13167642

TRANSITION GAMMA = 4.435177

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